TRINITY COLLEGE

Mathematics Methods Unit 3/4
Test 4 2022

Section 1 Calculator Free

Logarithms

STUDENT'S NAME _____

DATE: Thursday 30th June **TIME:** 25 minutes **MARKS**: 29

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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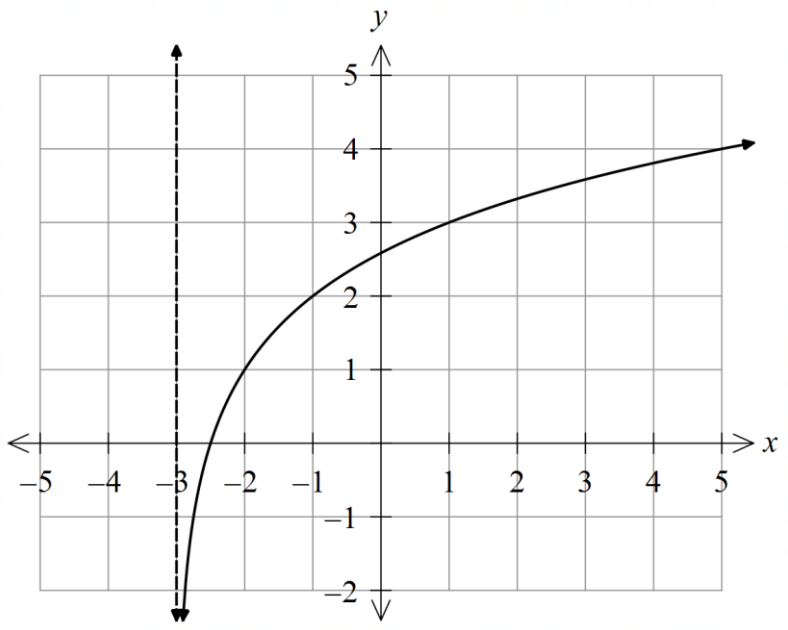
1. (3 marks)

Consider the function $y = f(x)$ graphed below.



Given $f(x) = \log_a(x-b)+c$, determine the values of a, b, and c.

2. (7 marks)

Differentiate each of the following with respect to x.

(a) $\ln(x^3 + 4x - 5)$ [1]

(b) $e^{2x} \ln(\sqrt{x} + 5)$ [2]

(c) $\ln\left(\dfrac{x-6}{(3x+5)^4}\right)$ [4]

3. (10 marks)

(a) Express the expression $2\log(4x+3)$ in terms of the natural logarithm.

[2]

(b) Solve exactly for x in each of the following equations.

(i) $\log_x 5 = 0.5$ [1]

(ii) $\log_4 x - \log_4(x+3) = -1$ [3]

(iii) $6^{x-1} = 2^{x+1}$ [4]

4. (9 marks)

(a) Evaluate $\int_1^2 \left(e^x + \frac{1}{x} \right) \, dx$. [3]

(b) If $\frac{dV}{dt} = \frac{(2t-1)(2t+1)}{t}$, determine V in terms of t, given that the function V passes through the point $(1,5)$. [4]

(c) Determine the antiderivative of $\frac{\pi}{\tan x}$ with respect to x. [2]

TRINITY COLLEGE

Mathematics Methods Unit 3/4
Test 4 2022

Section 2 Calculator Assumed

Logarithms

STUDENT'S NAME _____

DATE: Thursday 30th June **TIME:** 20 minutes **MARKS**: 20

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (3 marks)

pH is a measure of how acidic or alkaline a substance is, and the pH scale goes from 0 to 14, 0 being most acidic and 14 being most alkaline. Water in a stream has a neutral pH of about 7. The pH (p) of a substance can be found according to the formula $p = -\log h$ where h is the substances hydrogen ion concentration.

(a) A bottle of apple juice purchased has a hydrogen ion concentration of about $h = 0.0002$. Determine the pH of the apple juice, correct to one decimal place and hence state whether it is acidic or alkaline. [2]

(b) A banana has a pH of about 8.3. Determine the concentration of hydrogen ions, leaving your answer as an exact value. [1]

6. (8 marks)

An oil tanker is leaking at the rate $L'(t) = \frac{80\ln(t+1)}{t+1}$, where $L'(t)$ is hundreds of litres per hour and t is the number of hours after the leak occurs.

(a) Determine the initial rate of the leak. [1]

(b) Determine the total volume of oil that the ship will leak on:

(i) the first day. [2]

(ii) the second day. [1]

(c) Comment on the rate of the oil leak as t increases. [1]

(d) The leak is repaired after the oil tanker has spilled 150 kL of oil into the ocean. Determine how many days after the initial leak the oil tanker is repaired. [3]

7. (9 marks)

An object has a displacement function $s(t) = t - \ln(8t + 1)$ where s is in metres and t is in seconds. Determine:

(a) the initial position of the object. [1]

(b) the velocity function of the object. [1]

(c) at what time the object changes direction. [2]

(d) how far the object travels in the first 5 seconds. [3]

(e) at what time the object returns to the origin. [2]